Nikos Fokas, Ákos Kopper, Máté Maródi, and Gábor Szedenics

On the Fractal-Like Properties of National and Regional Railroad Networks in Europe[*] [1]

Introduction

Communication is ubiquitous. We send a great variety of signals to one another in different places and different situations – in our personal relations, for instance, when we try to get along a crowded bus without a word, stand moaning in a queue, or vehemently gesture in the relaxed atmosphere of a Sunday lunch or a coffeehouse conversation. But people also need intense communication to be able to create larger organizations and institutions. Ultimately, at the level of world-wide communication networks, Earth is a single global village, since Aristotle already knew that the boundaries of an empire can be drawn where the voice of the messenger gets in a hundred days.

In the most colloquial sense, any universe or set of people is obviously turned into a structured community with hierarchical organization by the communication networks which develop between its members. Therefore it may be the central objective of diverse trends in social science to understand the operation of various communication systems and subsystems, and to explore their structural properties.

We were interested in the nature of *structures* and attempted to utilize the results of studies concerning fractal sets accumulated in the past decade in the analysis of these structures. However, we had to face the permanent difficulty of applying findings which derive from the natural sciences in the domain of social science. Although theories and methods were more or less available, we had no data to verify them. How could we, with reasonable accuracy and according to the aims of quantitative fractal analysis, reconstruct the communication network of a Sunday lunch, a family, or, say, a scientific community? The idea that these methods should be used in analyzing maps came to our minds first when we read a study written by Lajos Nyikos and his associates (Nyikos, Balázs, and Schiller 1994). They digitized and analyzed the graphic images of various artists, for example, Picasso and Dürer, in terms of fractal geometry. Logically, we could start our investigation with the analysis of networks which can be represented well as images. In sociology images are usually *maps,* and if we try to find communication networks, then transport networks offer an obvious choice. In this way we had arrived at the transport network with the most definite boundaries: the railway system. There can be no doubt about its importance in social history. The great railroad construction projects which began in the middle of the past century in Europe provided the most important basis for extensive economic growth and served as the hallmark of the same. Many people think that there is a close relation between the beginning

[*] Acknowledgements: This research has been supported by the Hungarian Scientific Research Fund (OTKA) under project no. T20370. Thanks also to Lajos Nyikos for his assistance in launching this research, to Attila Vincze, who provided a fractal dimension calculator software developed by himself and helped us understand how it works, to Ágnes Medveczki and László Eperjesi for guidance in the unfamiliar world of railroad maps, to Mrs. Surányi for her assistance in photocopying the maps, and to Tamás Vicsek, who drew our attention to several studies by like-minded researchers.

[1] In Fokas N. (ed): Order and Chaos, Budapest, 1997, Replika Books, (in Hungarian)

of World War I and the construction of the Orient Express line reaching Baghdad, and thus the end of the great railroad construction period in Europe.

The Method

In addition to all other arguments, the analysis of railroad maps seemed to be the simplest way in technical terms too. We have maintained this opinion, although this "simplest" did not prove to be really easy. We relied mainly on the detailed Atlas der Eisenbahnen und Schiffahrt von Europa album, which was published in the 1930s. It was complemented with several dozens of large regional maps from the relevant division within the library of the Transport Museum of Budapest, published in the period between the 1910s and the 1930s, and in the 1950s, as well as with the 1979 World Atlas and 1991 Historical World Atlas of Cartographia Ltd. For the sake of more accurate measurement and experimentation, and with the aim of exploring the degree to which results are dependent on the map in question, maps of various scales were applied, for both the selected country and region.

Maps of all European countries, the Central European region, which is often defined in different ways, and the whole of Europe were available. It posed a specific problem that the size of these images varied from A3 to desktop, and that a given country was not only represented on several pages in an album, but scales used on these pages were also different in some cases. After drawing onto tracing paper, shrinking and fitting via an iterative process, some sixty A4-size maps were produced. The scanned versions of these maps were adjusted to the default maximum size of 700×700 pixels defined by the fractal dimension calculator software, and diverted from this value only when the fine details of the railroad network or the shape of the country in question required so.

When examining fractal properties, we also had to face a common problem: mathematically, fractals are objects which can be created through an *infinite* number of recursive steps, where a self-similar or self-affine nature has to result at *all* scales, but, of course, empirically observable objects do not meet this requirement. All we could say about Mandelbrot's famous example, the coast of Britain, is that this self-similarity appears across a wide but still *finite* range of scales. To reduce the uncertainty usually present in such empirical studies, two types of procedures were applied in defining the fractal dimensions of railroad networks, which evidently have similar properties in many respects (Tél 1988; Vicsek 1989; Schroeder 1991). Eventually, if we try to find out how the appearance of a structure is altered by change in scale, we can basically do two things: we can either approach from a greater distance and gradually explore the finer details, or vice versa, start at finer scales and gradually move away from the figure.

The first path is followed by the box-counting method, in which a grid with an increasing r resolution value is placed onto the map. Next, we examine how the $N(r)$ number of cells which contain at least one pixel depends on the changing resolution of this grid. For fractals, the condition $N(r) \propto r^{-D_b}$ should be met, where D_b is the box-counting dimension. The version of this method applied in our study set off from a grid with L5 cell size, where cell size equals one-fifth of the minimum page size L = min{(N,M)} for a map of N×M size in pixels, and in steps of one pixel at a time refined the grid up to the maximum resolution of two pixels per cell.

The other method for analyzing images, the sandbox procedure follows a path of the opposite direction. First a black pixel is defined as the origin, then the $M(r)$ number of pixels which can be found within a square with sides of length r drawn around the origin is defined.

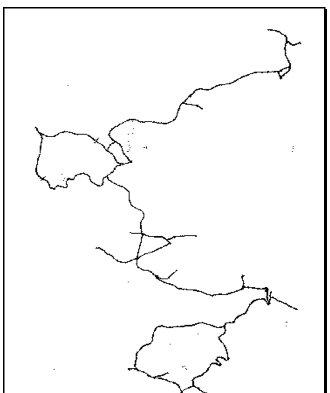
It is common knowledge that fractals meet the condition $M(r) \propto r^{-D_m}$, where D_m is the exponent of mass. This method is generally thought to be particularly sensitive to the potential inhomogeneity of the studied structure, and thus all pixels within a rectangle defined in the middle of the map were used as origin points in the course of the actual application. Starting from a minimum cell size of 6 pixels and approaching the maximum radius of 266 pixels in 4-pixel steps, the black pixels surrounding these origin points were counted. Finally, the data received through this procedure were averaged.

The Results

The findings of the research surpassed our expectations, since we are faced with much more unanswered questions today than in the beginning. Therefore this article is not intended for a final report but an initial brief account of the investigations which we have performed. Calmingly, very similar results were received via the different methods. Although the sandbox method, which is considered more reliable, systematically produced somewhat higher values, differences were significant in a few cases only. Variations did not show a systematic pattern, but further and more careful analysis will be needed to clarify this issue. Notably, the order of magnitude for fitting, which is so important for us, was typically and often significantly greater for the sandbox technique than the box-counting method. The reason for this phenomenon is currently unknown.

Based on our data, three typical categories of railroad networks can be distinguished.

Greece and Turkey fell into one group. As Maps 1 and 2 show, these countries have a rather sparse, line-like railroad network. Quantitative results confirm this impression, as far as we can claim it on the basis of the inaccurate fit (social scientists should note that here the value R^2=0.99 is considered inaccurate because good fit provides a value of R^2=0.999!), the value of the fractal dimension was close to 1 for both countries (see Diagrams 1 and 2).



Map 1: The Railroad Network of Greece

Map 2: The Railroad Network of Turkey

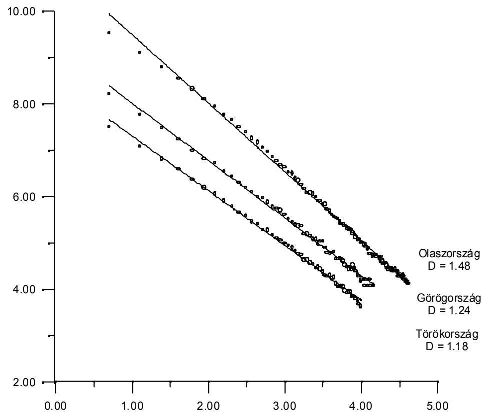


Diagram 1: The Box-Counting Method

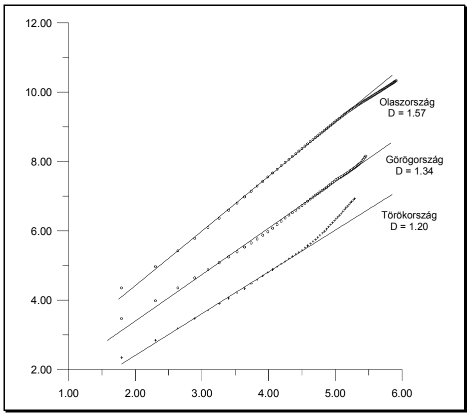
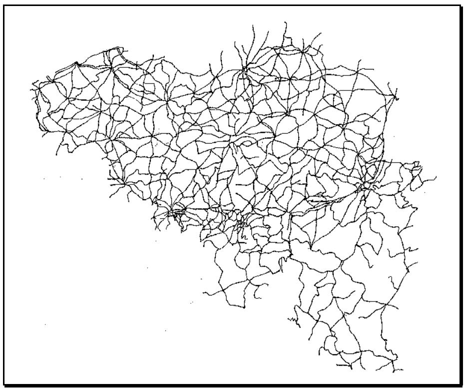
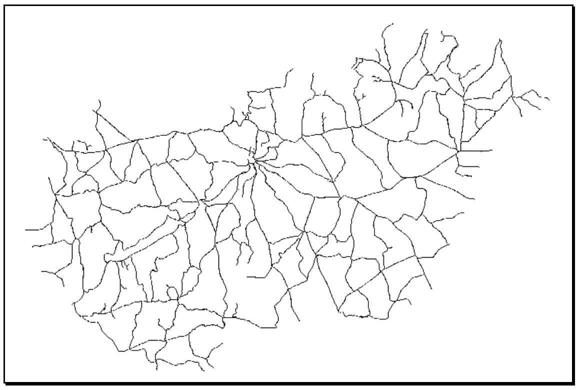


Diagram 2: The Mass Exponent

Italy was included in Diagrams 1 and 2 only to indicate transition to a much more populous category. This group contains, among others, the Netherlands, Belgium, as well as Spain and today's Hungary (see Maps 3 and 4).



Map 3: The Railroad Network of Belgium



Map 4: The Railroad Network of Hungary

The remarkable break in Diagrams 3 and 4 indicates that these railroad networks have different structural properties at finer and coarser scales. In the range of finer scales they show the same line-like behavior which characterized the entire Greek or Turkish networks. However, at coarser scales both methods produce fractal dimensions of approximately 1.75, which implies that these networks, constructed from straight lines, thoroughly fill the two-dimensional space available for them. We could say: Got it! – The problem is that the fit stands across one order of magnitude only, which lags behind the minimum two orders of magnitude generally expected by researchers performing empirical studies in the natural sciences.

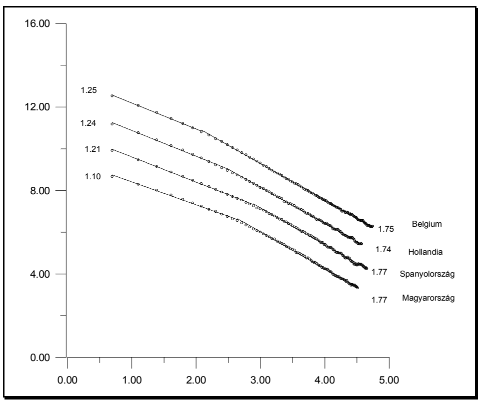


Diagram 3: Box-Counting

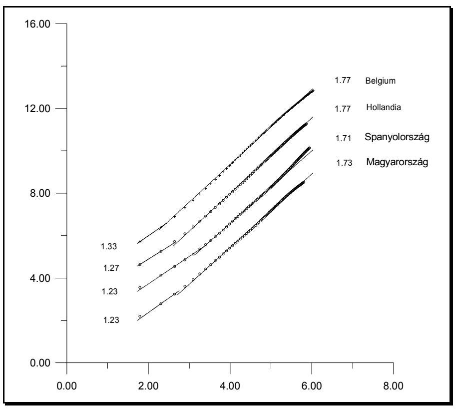
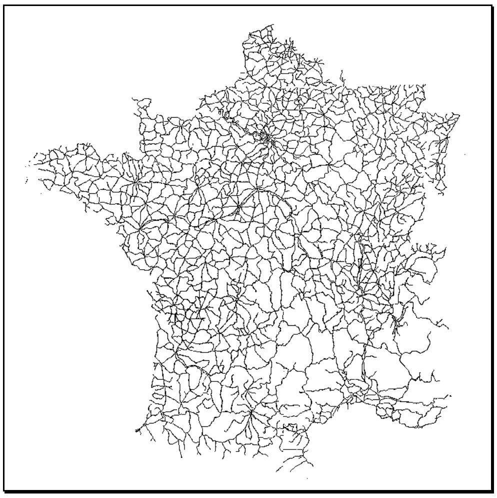
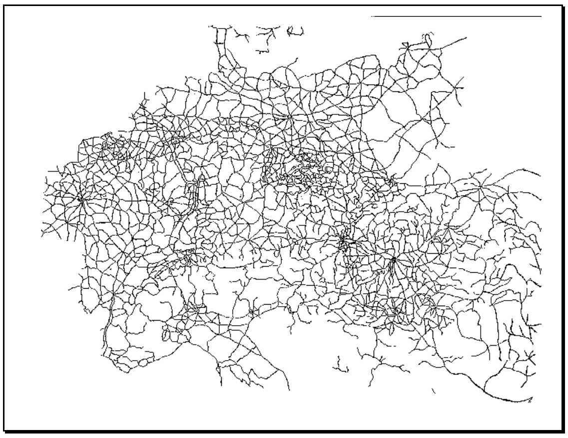


Diagram 4: Mass Exponent

In fact, we could not find a fit with this magnitude order but one which approximates it well. This category includes the railroad networks of France, Central Europe, the United Kingdom and former Germany (see Maps 5 and 6).

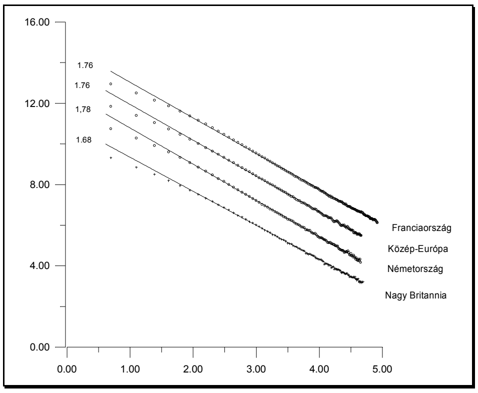




Map 5: The Railroad Network of France Map 6: The Railroad Network of Central Europe

Here the fit stands for almost the entire range of scales used in the map (as shown in Diagrams 5 and 6), and spans a size increase of almost 70 times. Assumably, we cannot expect fit of a coarser scale, since railways are typically used over a certain distance. We think that the magnitude order of fitting, particularly at finer scales, could be increased through gradually taking the road network into account.

It can be seen remarkably well in our diagrams that the fractal nature of the railroad network is determined by both the magnitude order of the fit and the value of the fractal dimension. However, it should also be noted that the sandbox and box-counting methods produced very similar dimension values (France may be an exception), and that this dimension slightly differs from the value of about 1.75 observed for the previous category.



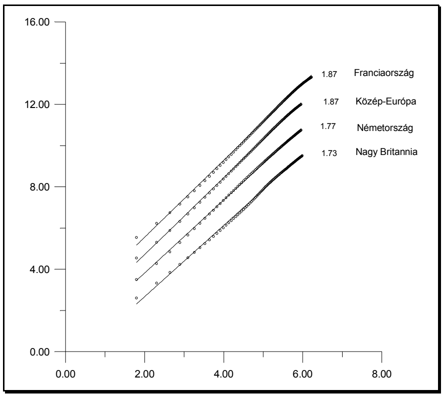
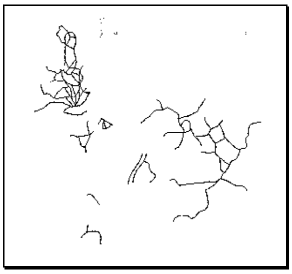
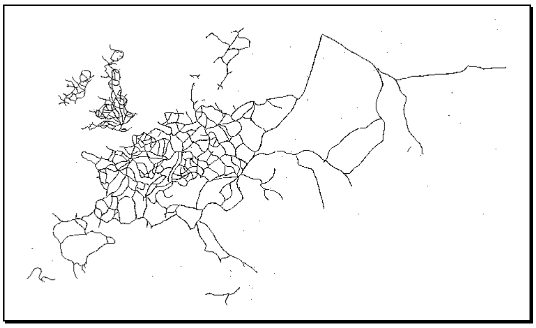


Diagram 5: Box-Counting Diagram 6: Mass Exponent

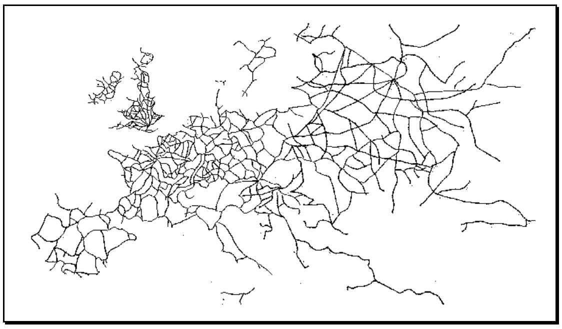
It should be also mentioned that the static results above hide an obvious historical dynamics. It is not to say that Turkey should see the present French railroad network as an example of its own future railway system. The age of railroad construction has ended in Europe. Nevertheless, if we analyze the development of either the European or the Hungarian networks over time (see Maps 7 to 9 and 10 to 14, respectively), the gradual emergence of fractal-like properties which can be observed here (see Diagrams 7 to 8 and 9 to 10, respectively) shows the same picture as the „growth" process for the networks of Turkey, Greece, the Netherlands or France (see Diagrams 11 and 12).



Map 7: The Railroad Network of Europe, 1850



Map 8: The Railroad Network of Europe, 1870



Map 9: The Railroad Network of Europe, 1914

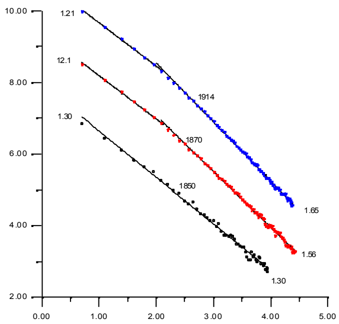


Diagram 7: Box-Counting, Europe

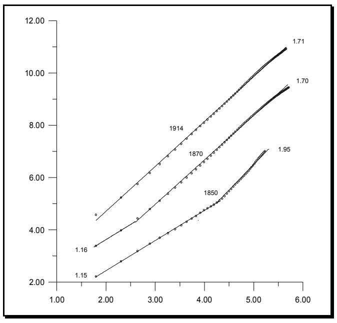
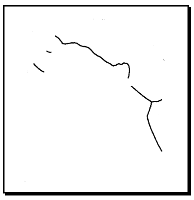
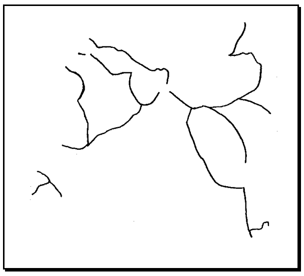


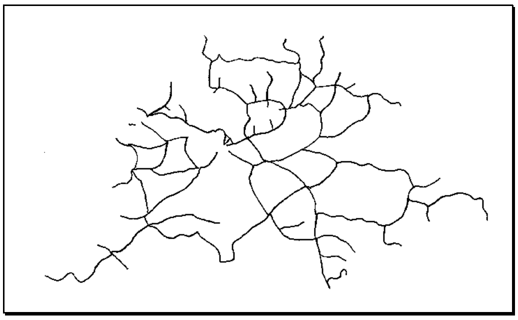
Diagram 8: Mass Exponent, Europe



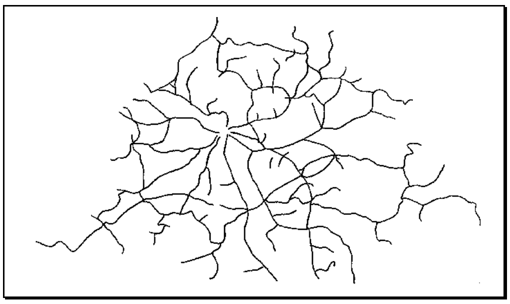
Map 10: The Railroad Network of Hungary, 1855

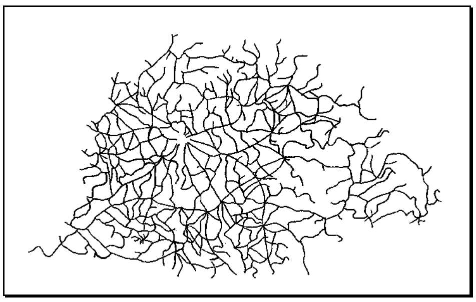


Map 11: The Railroad Network of Hungary, 1867

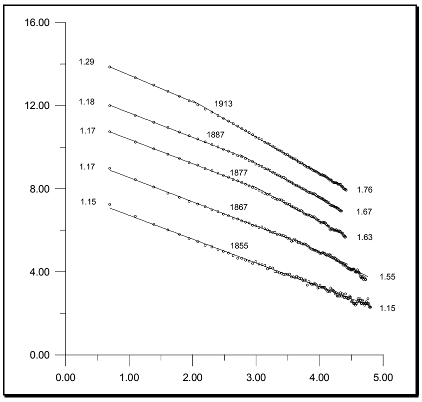


Map 12: The Railroad Network of Hungary, 1877





Map 13: The Railroad Network of Hungary, 1887 Map 14: The Railroad Network of Hungary, 1913



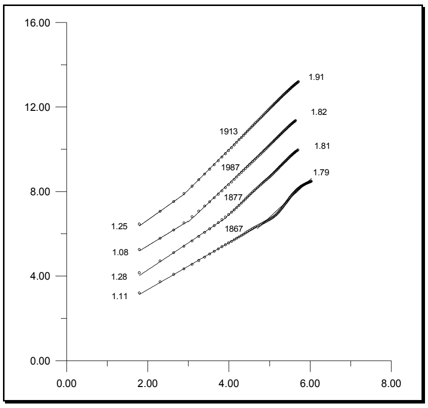
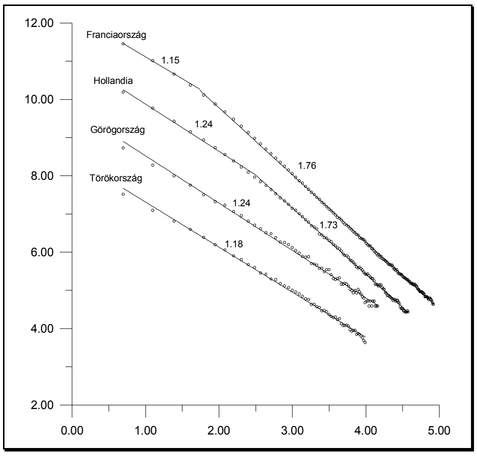


Diagram 9: Box-Counting, Hungary Diagram 10: Mass Exponent, Hungary



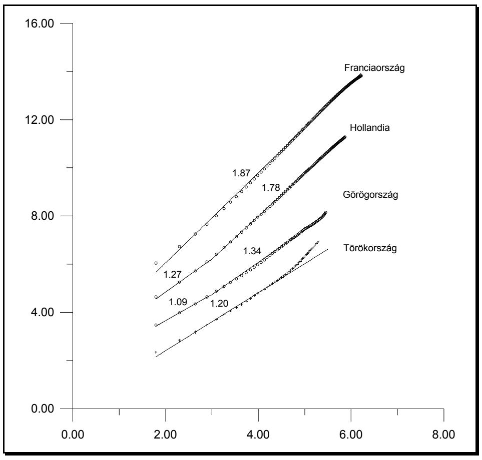


Diagram 11: Box-Counting Diagram 12: Mass Exponent

Based on the comparison of Diagrams 7 to 10 and 11 to 12, we might conclude that the initial development of railroad networks in selected European countries followed a growth path which resulted a fractal dimension value of about 1.75, unless the appearance of other competitive forms of transport, the lack of further sources of investment, or political changes, etc. halted the process.

Naturally, the conclusion above makes sense only if we can clarify the statement that the railroad networks of certain countries can be considered a fractal with a particular dimension. This question cannot be avoided, and we have not found an adequate answer to it. Below we will attempt to outline the scope for a potential answer, and also sketch the predicted major directions of further research.

Since it was Mandelbrot who discovered fractals, and following his classical work (Mandelbrot 1977), we generally speak of a new geometry of nature in relation to fractals, let us also start from the geometrical analysis of some very simple figures. We can take either a cube or a sphere, whose volume depends on the third power of linear sizes (side length or radius). Therefore a tenfold increase in the side length of a cube results a thousand-times increase in its volume, and if it is a homogeneous, compact figure, the same will happen to its mass. It is not very practical to generate large-size objects in this way. This method results an extraordinary loss of material and also poses the risk that the construction simply collapses when it exceeds a certain size because of its rapidly increasing weight. It is much more reasonable and economical to create spongy, porous figures, and to omit all excess materials in order to keep only the pure structure. This way of construction can be observed in nature as well as in objects made by humans, for instance, a tree crown and the Eiffel Tower.

However, the “punched” cube or sphere assumes a new, interesting property from another aspect too. In contrast with normal geometrical objects, the surface of objects with such structures can be very large in relation to their volume. It is frequently useful for a figure which occupies some limited space to have a large surface. Take, for instance, a catalyst or a filter, and the same conditions are fulfilled by human lungs, having a small volume but a surface of almost a hundred square meters. Obviously, a similar requirement can be set for all networks of communication, and thus very similar structural features can be observed for organs with very different functions.

What does it mean with respect to railroad networks? The requirement of small “mass”, to be understood figuratively, can mean here the principle of saving material and energy, in other words, the demand for a network which can be constructed as cheaply as possible, and can be maintained and operated with minimum expenditure. Therefore it would be an unrealistic objective to develop a railroad network which reaches *all* points of the country. However, the demand for having a “surface” as large as possible sets, in a somewhat contradictory way, the requirement that the network should get as *close* as possible to as *many* points of the country as possible. As the system of blood vessels or the respiratory system should fill the available space, i.e., the human body as well as possible, forming fractal-like structures (Goldberger, Rigney, and West 1991), railroad networks can also be expected to do the same within a given space.

Obviously, the railroad network of Hungary meets this requirement, since the area of the country equals that of a square having sides of about 305 kilometers, but railway lines with a total length of 7757 kilometers are “crammed” into this relatively small space. In contrast, the 112 thousand square kilometer area of Greece, calculated without the islands, holds only railway lines with a total length of 2479 kilometers. This difference clearly manifests in the value of fractal dimensions (D_{HU} =1.77 and D_{GR} =1.24, respectively), which supports the assumption that the more dimensions a fractal has, the more likely it is that a part of the space which includes the fractal will also include a part of this fractal.

However, the fractal properties of a railroad network is much more than line length per unit area. Fractals meet the above requirements: they are “spongy” and “porous” structures with a high “surface-volume” ratio, but they are also self-similar, i.e., they fill the available space identically across a wide range of scales (from “finer” to “coarser”), although they seem to be irregular. Consequently, the railroad network of a country, as far as it is a fractal, can be considered a hierarchical organization which shows non-trivial regularity. This feature obviously reflects the similar structured nature of the social, economic and power factors which shape the development of the railroad network. At the time of great railway constructions a provincial town or a local potentate could play the same role within their jurisdiction as, say, Budapest or a magnate family at the national level. However, now we have no accurate picture about the factors which formed the railroad network of a country and about how they did it. Findings from similar research projects show that, hopefully, we can find a model (of growth, perhaps) which explains the development of railroad networks – it will also be the adequate answer to the question what it means that these networks have fractal properties.

Further Questions

Reasonably, we can apply other methods of calculating fractal dimensions in order to extend and refine our findings. Perhaps it will help reveal whether results produced through various methods show systematic differences. Additional studies are needed to check how sensitive applied methods are to the finite size and varying resolution of maps.

Of course, our research can be extended to the study of railroad networks in other parts of the world. But even if we stay within Europe it is already clear that results show a specific regional distribution. It is enough to remember that the lowest values of the fractal dimension were found in South East Europe. Evidently, it supports the potential of extending this research over studying the historical regions of Europe. In our opinion, we could use the methods applied so far or their slightly modified versions (e.g., moving the grid) to discover regions which form an organic unit irrespective of country borders. We hope that it will bring many interesting results if the mass exponent is always defined through starting from a selected center. It will allow us to determine the "scope" of a given city, for example, Paris, London or Budapest, and also to interpret the development of a railroad network as a growth process. We plan something similar to the study of the underground network in Paris (Benguigui and Daoud 1991).

Finally, we hope to find a dynamical model describing the development of transport networks, which is similar to those applied in simulating the growth processes of cities (Makse, Havlin, and Stanley 1995) or the development of footpaths (Helbing, Keltsch, and Molnár 1997). The question is whether we can have a feasible "model of potential" where various settlements would be attractive centers, the strength of potential would indicate the importance or "appeal" of a settlement, and railway lines would appear at places with minimum potential values.

Conclusion

Perhaps it is unnecessary to say that we have never been, and have not become through this research, railway experts. Of course, the analysis of the railroad network has proved to be an interesting problem which deserves further studying. However, apparently there is no obstacle to extending this procedure, if proper data are available, over any other indicators which are relevant for social history and sociology, and can be recorded on maps. For example, it is well known that the historical regions of Europe themselves can be analyzed through a great variety of maps – "maps which would depict, say, the spread of Romanticism and Gothicism, or Renaissance and Reformation; and even maps which, for instance, highlight autonomous cities, corporative freedoms, estate organizations and a series of other … structural features" (Szűcs 1981). It also promises interesting results to study the diffusion of literacy, modern universities, book publishing, certain consumption habits, cultural patterns or diseases (such as the plague in the Middle Ages or AIDS today), or technical innovations (such as the plow in the early modern age or the Internet in our century), and to analyze various structures of towns or other settlements and transport networks. We could go on listing additional examples but it would be pointless, since the actual decisions will be made by the specialists of the different fields.

List of Maps

Map 1: Atlas der Eisenbahnen und Schiffahrt von Europa.
Map 2: Világatlasz (World Atlas). Budapest: Cartographia Ltd., 1979. Pp. 86–87.
Map 3: Atlas der Eisenbahnen und Schiffahrt von Europa.
Map 4: Világatlasz (World Atlas). Budapest: Cartographia Ltd., 1979. Pp. 34–35, 38–39, 42–43, 45–47.
Map 5: Atlas der Eisenbahnen und Schiffahrt von Europa.
Map 6: Közép-Európa vasuti térképe, 1914–1918 (The Railway Map of Central Europe, 1914–1918). Károly Lajos Posner and Son.
Maps 7 to 9: Történelmi Világatlasz (Historical World Atlas). Budapest: Cartographia Ltd., 1991. P. 63.
Maps 10 to 14: Történelmi Világatlasz (Historical World Atlas). Budapest: Cartographia Ltd., 1991. P. 129.

References

Benguigui, L., and M. Daoud (1991). "Is the suburban railway system a fractal?" *Geographical Analysis* 23: 362–368.

Goldberger, A.L., D. Rigney, and B.J. West (1991). "Káosz és fraktálok az emberi szervezetben (Chaos and Fractals in the Human Organism)." *Tudomány* (?): 29–35.

Helbing, D., J. Keltsch, and P. Molnár (1997). "Modelling the evolution of human trail systems." *Nature,* vol. 388, 3: 47–49.

Makse, H.A., Sh. Havlin, and H.E. Stanley (1995). "Modelling urban growth patterns." *Nature,* vol. 377, 19: 608–612.

Mandelbrot, B. (1977). *The Fractal Geometry of Nature*. New York: W.H. Freeman and Co.

Nyikos, L., L. Balázs, and R. Schiller (1994). "Fractal Analysis of Artistic Images: From Cubism to Fractalism." *Fractals*, vol. 2,1: 143–152.

Schroeder, M. (1991). *Fractals, Chaos, Power Law*. New York: W.H. Freeman and Co. Pp. 213–223.

Szűcs, J. (1981): "Vázlat Európa három történeti régiójáról (The Three Historical Regions of Europe: An Outline)." *Történelmi Szemle* 3: 313–359.

Tél, T. (1988). "Fractals, Multifractals, and Thermodynamics." *Z. Naturforsch* 43a: 1154–1174.

Vicsek, T. (1989): *Fractal Growth Phenomena*. Singapore: World Scientific Publishing Co. Pp. 9–55.